Calculus 2 — Quiz 1

Scope: Chapter 8, Chapter 9 (Sec. 7–10), Chapter 10

Name: ______________________________ Student ID: ______________________

Show all work clearly. Answers without justification receive no credit. Calculators are NOT permitted.

1. (12%) Evaluate the following integrals.

(a) (6%) $\int x \arctan x \, \mathrm{d}x$

(b) (6%) $\int e^x \sin x \, \mathrm{d}x$

2. (10%) Evaluate the following integrals.

(a) (5%) $\int \cos^5 x \, \mathrm{d}x$

(b) (5%) $\int \sec^4 x \tan^2 x \, \mathrm{d}x$

3. (10%) Evaluate the following integrals.

(a) (5%) $\int_0^1 x^2\sqrt{4-x^2} \, \mathrm{d}x$

(b) (5%) $\int \frac{e^x}{e^{2x}+3e^x+2} \, \mathrm{d}x$

4. (10%) Evaluate each improper integral, or show it diverges.

(a) (5%) $\int_1^\infty xe^{-x} \, \mathrm{d}x$

(b) (5%) $\int_0^1 \ln x \, \mathrm{d}x$

5. (12%)

(a) (4%) Find the 4th-degree Maclaurin polynomial $P_4(x)$ for $f(x) = xe^x$.

(b) (8%) Find the Maclaurin series for $f(x) = \frac{x}{1+x^2}$.

6. (14%)

(a) (6%) Use Maclaurin series to evaluate

$$\lim_{x \to 0} \frac{x - \sin x}{x - \arctan x}.$$

(b) (8%) Starting from the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $(|x| < 1)$, differentiate term by term to find a power series for $\frac{1}{(1-x)^2}$.

7. (14%)

(a) (6%) Consider the cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$.

(i) Find $\frac{dy}{dx}$ in terms of θ.

(ii) Find all $\theta \in (0, 2\pi)$ where the tangent line is horizontal.

(b) (8%) Find the arc length of the curve $x = e^t \cos t, \quad y = e^t \sin t \quad \text{for } 0 \le t \le \pi$.

8. (18%)

(a) (4%) Convert the polar equation $r^2 = \cos(2\theta)$ to rectangular form.

(b) (6%) Find the slope of the tangent line to $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

(c) (8%) Find the area of the region that lies inside the circle $r = 4\cos\theta$ and outside the circle $r = 2$.

Function	Taylor series	Interval of convergence
$\frac{1}{x}$	$1-(x-1)+(x-1)^2-(x-1)^3+\cdots.+(-1)^n(x-1)^n+\cdots.$	$0<x<2$
$\frac{1}{1+x}$	$1-x+x^2-x^3+\cdots.+(-1)^n x^n+\cdots.$	$-1<x<1$
$\ln x$	$(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\cdots.+\frac{(-1)^{n-1}(x-1)^n}{n}+\cdots.$	$0<x\le 2$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots.+\frac{x^n}{n!}+\cdots.$	$-\infty<x<\infty$
$sin(x)$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots.+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots.$	$-\infty<x<\infty$
$cos(x)$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots.+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots.$	$-\infty<x<\infty$
$arctan(x)$	$x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots.+\frac{(-1)^n x^{2n+1}}{2n+1}+\cdots.$	$-1\le \mathrm{x}\le 1$
$arcsin(x)$	$x+\frac{x^3}{2\times 3}+\frac{1\times 3x^5}{2\times 4\times 5}+\cdots.+\frac{(2n)!\,x^{2n+1}}{(2^n n!)^2(2n+1)}+\cdots.$	$-1\le \mathrm{x}\le 1$
$(1+x)^k$	$1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}$ $+\cdots.+\frac{k(k-1)\ldots(k-n+1)x^n}{n!}+\cdots.$	$-1<x<1$

Derivative	Integrals
$\frac{d\sin^{-1}u}{dx}=\frac{u'}{\sqrt{1-u^2}}$	$\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\frac{u}{a}+C$
$\frac{d\,cos^{-1}u}{dx}=\frac{-u'}{\sqrt{1-u^2}}$	$\int\frac{du}{a^2+u^2}=\frac{1}{a}tan^{-1}\frac{u}{a}+C$
$\frac{d\tan^{-1}u}{dx}=\frac{u'}{1+u^2}$	$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}sec^{-1}\frac{\lvert u\rvert}{a}+C$
$\frac{d\,cot^{-1}u}{dx}=\frac{-u'}{1+u^2}$	
$\frac{d\sec^{-1}u}{dx}=\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	
$\frac{d\,csc^{-1}u}{dx}=\frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$	